harmonic of supersaturation perturbation; A, amplitude of highly nonlinear self-excited oscillations; G, Gibbs number; m_k, k-th moment of crystal-size distribution function. The index $^\circ$ signifies that the corresponding quantity is evaluated on the surface of neutral stability; angle brackets denote time average; the asterisk denotes complex conjugates.

LITERATURE CITED

1. Yu. A. Buevich, V. V. Mansurov, and I. A. Natalukha, Inzh.-Fiz. Zh., 49, No. 2, 233-242 (1985).
2. S. S. Kutateladze, Similarity Analysis and Physical Models [in Russian], Novosibirsk (1986).
3. I. V. Melikhov and L. B. Berliner, Teor. Osn. Khim. Tekhnol., 12, No. 1, 48-53 (1978).
4. L. B. Berliner and V. N. Gorin, Teor. Osn. Khim. Tekhnol., 7, No. 5, 643-650 (1973).
5. V. V. Kafarov, I. N. Dorokhov, and Le Suan Khai, Dokl. Akad. Nauk SSSR, 267, No. 1, 147-150 (1982).
6. M. B. Sherwin, S. Shinnar, and S. Kats, AIChE. J., 13, No. 6, 1141-1154 (1967).
7. A. D. Randolph and M. A. Larson, Theory of Particulate Processes, New York (1971).
8. J. Garside, Chem. Eng. Sci., 40, No. 1, 3-26 (1985).
9. J. R. Bourne and M. Zabelka, Chem. Eng. Sci., 35, No. 4, 533-541 (1980).
10. B. Anshus and E. Ruckenstein, Chem. Eng. Sci., 28, No. 2, 501-513 (1973).
11. Yu. B. Ponomarenko, Prikl. Mat. Mekh., 28, No. 4, 688-698 (1964).
12. K. H. You and J. M. Douglas, AIChE J., 21, No. 5, 917-924 (1975).
13. S. Ul'm, Izv. Akad. Nauk Est. SSR, Ser. Fiz.-Mat. Nauk, No. 3, 115-131 (1965).
14. Yu. A. Buevich, V. V. Mansurov, and I. A. Natalukha, in: Mathematical Modeling in Science and Technology: Abstr. All-Union School-Seminar, Perm (1986), pp. 55-56.

VIBRATIONS AND NONUNIFORM HEATING OF A SHAFT IN A RADIAL BEARING

Yu. A. Buevich and M. I. L'vov

UDC 532.516

The article examines the problem of vibrations of the axis of a shaft in a radial bearing due to the imbalance of the system as a result of nonuniform heating of the shaft.

Trouble-free operation, longer life, and reliability of turbine rotors and rotating units of other installations require careful balancing with the object of minimizing the level of perturbing forces and moments acting on the plant. In practice it is impossible to eliminate these fluctuations completely; as a result, the axis of rotation, and also the axes of bearing shafts do not take up a strictly fixed position, instead they vibrate [1]. If the random effects on the system are negligibly small, then to such vibrations there correspond periodic motions of the point of intersection of the shaft axis with the plane of the bearing along some closed path, usually close to elliptical [2]. When the regime of rotation changes, it is possible that the characteristic linear dimension of the path (the vibration amplitude) abruptly changes; this prevents the normal functioning of friction units and may even lead to their destruction, or even to the breakdown of the installation itself or some of its parts [3]. It is therefore of interest to find the causes of such vibrations and the dependences of their characteristics on the physical and regime parameters.

One of the causes of imbalance of some plant (which would be perfectly balanced under isothermal conditions) may be the bending of shafts in bearings due to their thermal

A. M. Gor'kii Ural State University, Sverdlovsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 2, pp. 295-304, February, 1988. Original article submitted October 22, 1986.

0022-0841/88/5402-0214$12.50

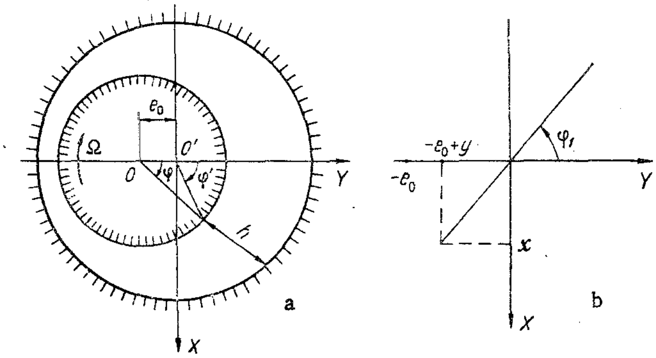

Fig. 1. Schematic illustration of the shaft in a bearing.

warping when they are heated nonuniformly over their section. If the shaft axis vibrates regularly with the notational frequency of the shaft, then, as follows from the theory presented below, certain parts of its surface will on an average exist under conditions that are most favorable for their heating by heat liberated in the layer of lubricant by viscous dissipation. This leads to nonuniform deformation of the shaft as a result of thermal expansion and its warping, which causes imbalance of the entire rotating system and finally gives rise to a centrifugal force acting on the shaft. This last is the cause of vibrations of the shaft relative to some mean position corresponding to ideal balance. Such a pattern is in fairly good agreement with many observed facts [4].

We emphasize that when the vibrations of the shaft are not synchronized with its rotation, the above-mentioned effect of nonuniform heating of the shaft and its deformation does not take place. Regardless of the obvious nonuniformity of the steady-state temperature distribution in the layer of lubricant of the loaded bearing (which may be increased even more when the viscosity of the liquid decreases substantially with rising temperature), all parts of the surface of the rotating shaft are subjected to exactly the same conditions since they in turn come into contact with all the parts of the layer of lubricant in the course of each period.

Strict quantitative description requires the simultaneous solution of mutually connected nonlinear problems of hydrodynamics, thermal conductivity, and thermoelasticity in regions of complex geometry with the imposition of conditions, many of which are determined entirely by the actual organization of the friction units, design features of the plant, etc. [5]. To obtain the reviewed results of a fairly general nature, we deal below solely with a simplified model of an ideal radial bearing; within its framework we can then divide and linearize various boundary-value problems. Such results are important in the sense that they make it possible to point out fundamental methods of analyzing the above-mentioned phenomena.

The Effect of Shaft Vibrations on Flow in the Layer of Lubricant. We deal with a radial bearing whose cross section is illustrated in Fig. 1. In investigating the flow in the layer of lubricant we neglect the dependence of the viscosity of the liquid on the pressure and temperature, and we use Reynolds' ordinary approximation, i.e., we take it that $h/R \sim d/R \ll 1$ and $\Omega RH/\nu \ll 1$ and we take into account only the terms of zeroth order with these small parameters. Then we have the equations [6]:

$$\frac{1}{R}\frac{\partial p}{\partial \varphi} = \mu\frac{\partial^2 u}{\partial r^2},\ \frac{\partial p}{\partial r} = 0,\ \frac{1}{R}\frac{\partial u}{\partial \varphi} + \frac{\partial w}{\partial r} = 0 \tag{1}$$

(it is easy to verify that the smallness of the ratio h/R also makes it not possible to distinguish between the coordinates r, φ and r', φ', associated with the points 0 and 0', respectively, in Fig. 1).

Assume that the axis of the shaft oscillates so that the coordinates of point 0 are equal to $x(t)$, $-e_0 + y(t)$, where e_0 is the eccentricity corresponding to the unperturbed force $\mathbf{P}_0$ applied to the shaft and directed along the X-axis. We take it that x and y are small compared with e_0; therefore the following parameters (see Fig. 1) are also small:

$$\varphi_1 = \arcsin(x/e) = x/e_0,\ e_1 = e - e_0 = -y. \tag{2}$$

For the sake of convenience we multiply all small values additionally by the factor ε which at the end of the calculations has to be put equal to unity. Confining ourselves to the analysis of the effects of distortions of the first order, we retain in the calculations solely the terms of zeroth and first order of powers of ε.

The thickness of the clearance between the shaft and the bearing is written with sufficient accuracy in the form

$$h = d + e\cos(\varphi + \varepsilon\varphi_1) = h_0 + \varepsilon h_1, \qquad h_0 = d + e_0\cos\varphi,\ h_1 = -x\sin\varphi - y\cos\varphi. \tag{3}$$

The tangential and the normal components of the speed of the points on the shaft surface can be represented as follows:

$$v_t = \Omega R + \varepsilon\left(\frac{dx}{dt}\cos\varphi - \frac{dy}{dt}\sin\varphi\right), \qquad v_n = \varepsilon\left(\frac{dx}{dt}\sin\varphi + \frac{dy}{dt}\cos\varphi\right),\ \Omega = \Omega_0 + \varepsilon\Omega_1 \tag{4}$$

(as positive we took the directions of unperturbed rotation of the shaft and of the vector of the normal that is the inner one in relation to the layer of lubricant).

The solution of the first equation (1) with the conditions $u = v_t(r = R)$ and $u = 0$ $(r = R + h)$ has the form

$$u = \frac{1}{2\mu R}\frac{\partial p}{\partial\varphi}(n - h)n + v_t\left(1 - \frac{n}{h}\right),\ n = r - R. \tag{5}$$

Hence, taking (4) into account, we obtain for the flow rate in the layer of lubricant that

$$Q = \int_0^h u\,dn = -\frac{h^3}{12\mu R}\frac{\partial p}{\partial\varphi} + \frac{h}{2}\left[\Omega R + \varepsilon\left(\frac{dx}{dt}\cos\varphi - \frac{dy}{dt}\sin\varphi\right)\right]. \tag{6}$$

On the other hand, from the last equation (1) follows:

$$\frac{1}{R}\frac{\partial Q}{\partial\varphi} = -w|_{n=h} + w|_{n=0} = v_n,$$

i.e., with (4) taken into account, we have

$$Q = Q_* + \varepsilon R\left(-\frac{dx}{dt}\cos\varphi + \frac{dy}{dt}\sin\varphi\right),\ Q_* = \frac{\Omega R h_*}{2}, \tag{7}$$

where Q_* (or h_*) plays the role of integration constant.

Equating (6) and (7), we obtain an equation for the pressure in the layer of lubricant (again we neglect terms of the order h/R)

$$\frac{\partial p}{\partial\varphi} = \frac{6\mu R^2}{h^3}\left[\Omega(h - h_*) + 2\varepsilon\left(\frac{dx}{dt}\cos\varphi - \frac{dy}{dt}\sin\varphi\right)\right], \tag{8}$$

from which follows

$$p = \mathrm{const} + 6\mu R^2\left[(J_3 - h_* J_2)\Omega + 2\varepsilon\left(J_{c,3}\frac{dx}{dt} - J_{s,3}\frac{dy}{dt}\right)\right]. \tag{9}$$

Here and below we introduce the designations:

$$J_n = \int_0^\varphi \frac{d\varphi}{h^n},\ J_{c,n} = \int_0^\varphi \frac{\cos\varphi\,d\varphi}{h^n},\ J_{s,n} = \int_0^\varphi \frac{\sin\varphi\,d\varphi}{h^n},$$

$$\{L_n,\ L_{c,n},\ L_{s,n}\} = \{J_n,\ J_{c,n},\ J_{s,n}\}_{\varphi=2\pi}.$$

The value of h_* is found from the requirement of periodicity p from (9) as functions of φ with the period 2π, from which it follows that

$$h_* = \frac{L_2}{L_3} + \frac{2\varepsilon}{\Omega}\frac{L_{c,3}}{L_3}\frac{dx}{dt}, \tag{10}$$

which finally closes the formulas presented above. We note that in these formulas the terms with ε due to perturbations are not only contained explicitly but also through the magnitude h determined in (2). For $\varepsilon = 0$ the obtained solution coincides with the classical one [6]. This solution makes it possible to describe flow in the layer of lubricant for any arbitrary movement of the shaft axis with small x and y.

Dynamic Response of the Shaft to the Perturbation of the Force and Moment Applied to It. We will now deal with the problem whose solution enables us to correlate the characteristics of the path of the shaft axis (i.e., the functions x(t) and y(t)) with the perturbations $\mathbf{P}_1$ and $\mathbf{M}_1$ of the force and the moment acting on the shaft from the side of the external sources. Introducing the mass m and the moment of inertia I in the calculation per unit length of the shaft (these values, after all, characterize not only the shaft but all the parts rotating with it), we write the equations of Newton's second law

$$md^2x/dt^2 = P_{1x} + F_{1x},\ md^2y/dt^2 = P_{1y} + F_{1y},\ Id\Omega_1/dt = M_1 + N_1, \tag{11}$$

where $\mathbf{F}_1$ and N_1 are small fluctuations of the force and moment acting on the shaft from the side of the layer of lubricant relative to their unperturbed values $(F_{0x} = -P_0,\ F_{0y} = 0,\ N_0 = -M_0$. For determining these values, and consequently also for closing the system (11) we use the results of the preceding chapter. With the same accuracy as before (i.e., neglecting values of the order of h/R), taking (1), (4), (5), and (8) into account, we obtain the expressions for the stresses acting on the shaft surface:

$$\sigma_{r\varphi} = \mu\,(\partial u/\partial n)_{n=0} = -\mu R\Omega\,\frac{4h - 3h_*}{h^2} - \frac{6\varepsilon\mu R}{h^2}\left(\frac{dx}{dt}\cos\varphi - \frac{dy}{dt}\sin\varphi\right),$$

$$\sigma_{rr} = -p + 2\mu\frac{\partial w}{\partial n}\bigg|_{n=0} = -p - \frac{2\mu}{R}\left(v_n + \frac{\partial u}{\partial\varphi}\right)_{n=0} = -p \tag{12}$$

(here again we take the smallness of h/R into account).

The moment of forces acting on unit shaft length and induced by the viscous stresses is equal to

$$N = N_0 + \varepsilon N_1 = R^2\int_0^{2\pi}\sigma_{r\varphi}d\varphi = N^* - 6\mu R^3 L_{c,2}\frac{dx}{dt},$$
$$N^* = -\mu R^3(4L_1 - 3h_* L_2)\,\Omega. \tag{13}$$

Using (8), we obtain with the same accuracy as before expressions for the components of the force acting on the shaft from the side of the layer of lubricant, calculated per unit shaft length:

$$F_x = F_{0x} + \varepsilon F_{1x} = R\int_0^{2\pi}(\sigma_{rr}\sin\varphi + \sigma_{r\varphi}\cos\varphi)\,d\varphi = -R\int_0^{2\pi}\frac{\partial p}{\partial\varphi}\cos\varphi d\varphi = F_x^* - 12\varepsilon\mu R^3 K_c\frac{dx}{dt},$$

$$F_x^* = -6\mu R^3(L_{c,2} - h_* L_{c,3})\,\Omega,$$

$$F_y = \varepsilon F_{1y} = R\int_0^{2\pi}(\sigma_{rr}\cos\varphi - \sigma_{r\varphi}\sin\varphi)\,d\varphi = R\int_0^{2\pi}\frac{\partial p}{\partial\varphi}\sin\varphi\,d\varphi =$$
$$= F_y^* - 12\varepsilon\mu R^3 K_s\frac{dy}{dt},\quad F_y^* = 6\mu R^3(L_{s,2} - h_* L_{s,3})\,\Omega. \tag{14}$$

Thus the values of (13) and (14) are represented in the form of the sums of two terms. The second term depends on the speed at which the shaft axis moves. The first terms, marked by an asterisk on top, have by their structure the same accuracy as in the corresponding unperturbed problem if we replace in the latter e_0 and Ω_0 by e and Ω, respectively, and take into account that the direction of the force F^* is determined by the angle φ, which is equal to $\pi/2-\varphi_1$ (and not $\pi/2$ as in the unperturbed problem). With the adopted accuracy we therefore have: $F_x^* = F^* \cos\varphi_1 = F^*$, $F_y^* = F^* \sin\varphi_1 = F^*\varphi_1$. The expressions for N* and F* are [6]:

$$N^* = -M^*, \; M^* = 4\pi\mu R^3\Omega \frac{d^2+2e^2}{(2d^2+e^2)\sqrt{d^2-e^2}},$$

$$\mathbf{F}^* = -\mathbf{P}^*, \; P^* = 12\pi\mu R^3\Omega \frac{e}{(2d^2+e^2)\sqrt{d^2-e^2}}. \tag{15}$$

Taking into account the smallness of the perturbations, and with a view to the abovesaid, we obtain:

$$\begin{aligned} N^* &= -M_0 - \varepsilon\left[\left(\frac{\partial M^*}{\partial\Omega}\right)_0 \Omega_1 + \left(\frac{\partial M^*}{\partial e}\right)_0 e_1\right], \\ F_x^* &= -P_0 - \varepsilon\left[\left(\frac{\partial P^*}{\partial\Omega}\right)_0 \Omega_1 + \left(\frac{\partial P^*}{\partial e}\right)_0 e_1\right], \\ F_y^* &= -\varepsilon P_0\varphi_1, \end{aligned} \tag{16}$$

where the factor ε was introduced in its adopted quality, and the subscript zero denotes that the corresponding derivatives are calculated in the unperturbed state (i.e., for $\Omega = \Omega_0$ and $e = e_0$).

From (13), (14), and (16), with (2) taken into account, we finally obtain:

$$\begin{aligned} N_1 &= -6\mu R^3 (L_{c,2})_0 \frac{dx}{dt} + \left(\frac{\partial M^*}{\partial e}\right)_0 y - \left(\frac{\partial M^*}{\partial\Omega}\right)_0 \Omega_1, \\ F_{1x} &= -12\mu R^3 (K_c)_0 \frac{dx}{dt} + \left(\frac{\partial P^*}{\partial e}\right)_0 y - \left(\frac{\partial P^*}{\partial\Omega}\right)_0 \Omega_1, \\ F_{1y} &= -\frac{P_0}{e_0} x - 12\mu R^3 (K_s)_0 \frac{dy}{dt}. \end{aligned} \tag{17}$$

In (14) and (17) we used the designations:

$$K_c = \int_0^{2\pi} \frac{\cos^2\varphi}{h^3}\, d\varphi, \quad K_s = \int_0^{2\pi} \frac{\sin^2\varphi}{h^3}\, d\varphi,$$

and the subscript zero denotes again that the corresponding values are calculated for the unperturbed state.

When we substitute (17) into (11), we obtain an inhomogeneous linear system of ordinary differential equations whose full order is equal to seven. Here the right-hand parts P_{1x}, P_{1y}, and M_1 have to be regarded as known functions of time.

If we are concerned solely with the effect of the centrifugal force due to imbalance, then with the adopted accuracy

$$P_{1x} + iP_{1y} = Ae^{i\Omega t} = Ae^{i\Omega_0 t}, \; M_1 = 0, \tag{18}$$

where A is the complex amplitude which, after all, can be made real by the appropriate choice of the instant from which time is measured. Finding the solution of system (11) in the form

$$x = \mathrm{Re}\,(B_x e^{i\Omega_0 t}), \; y = \mathrm{Re}\,(B_y e^{i\Omega_0 t}), \; \Omega_1 = \mathrm{Re}\,(B_\Omega e^{i\Omega_0 t}),$$

we can easily obtain by the standard method a system of linear algebraic equations whose solution expresses the complex amplitudes B_x, B_y, and B_Ω as magnitudes that are proportional to A from (18). When we furthermore eliminate the magnitude $\Omega_0 t$ from the expressions for x and y, we easily obtain the equation of the path of point O in Fig. 1 which in the approximation under consideration is an ellipse. The parameters of this ellipse (its principal semiaxes and the angle of slope of one of them to some coordinate axis) can then be easily determined from the mentioned equation.

The corresponding calculations are elementary but very cumbersome, and therefore we do not present them here. We note, however, that the elliptical paths of the shaft, observed directly on the screen of the oscillograph for a number of model and industrial systems with known centrifugal force, are in perfectly satisfactory agreement with the paths obtained theoretically on the basis of the above-explained linear theory.

We note that the importance of the system (11) with the magnitudes F_{1x}, F_{1y}, N_1 from (17) is not confined to the case of nonuniformity being due to perturbations in the form of (18). Thus the homogeneous system (11) is useful for the utilization of the hydrodynamic stability of a radial bearing [7]. Yet in distinction to a number of known approaches to the mentioned problem, with the theory developed by us there is no need to introduce a priori the coefficients of rigidity, compliance, friction of the system under consideration, since from (17) there follow perfectly concrete notions concerning these coefficients. The inhomogeneity of system (11) makes it possible to describe the dynamic response of the radial bearing to arbitrary small external perturbations of the force and moment. In particular, P_1 and M_1 can be random functions of time with known spectral characteristics. Then with the aid of the correlation theory of steady-state random processes it is easy to find from (11) and (17) the spectral characteristics of the magnitudes x, y, and Ω_1, i.e., to describe the response of the radial bearing to stochastic excitation, which is a very important problem.

In the context of the present work it is important that the obtained results make it possible to correlate x, y, and Ω_1, and consequently also perturbations of flow in the layer of lubricant, with the amplitude A of the centrifugal force. Below we will use the alternative notions

$$x = C_x \cos(\Omega_0 t + \beta_x),\ y = C_y \cos(\Omega_0 t + \beta_y),\ \Omega_1 = C_\Omega \cos(\Omega_0 t + \beta_\Omega), \tag{19}$$

regarding the coefficients C and the phase angles β as known magnitudes that depend on the physical and regime parameters of the system.

<u>Temperature Distribution over the Section of the Shaft.</u> The temperature fields in the shaft, in the layer of lubricant, and in the bearing are determined by the solution of the problem of convective heat conduction for these regions under conditions of continuity of the temperature and of the normal component of the heat flux on their boundaries. In the layer of lubricant there occurs heat release caused by viscous energy dissipation, with a density that is approximately equal to $g = (\mu/2)(\partial u/\partial n)^2$. In formulating the mentioned problem we also have to take into account the existence of heat sinks; this implies that we have to go to some extent beyond the limits of the plane problems of the type dealt with above. In limit situations heat may be removed through the shaft (and possibly also through the bearing) or from the layer of lubricant together with the lubricant itself. In the former case the decisive role is played by molecular heat conduction in the direction normal to the plane of the drawing, in the latter case by the flow of liquid in this direction. In both cases the result is very strongly dependent on design features of the system, and it can hardly be expected that conclusions of sufficiently general nature applicable to different systems will be obtained. For the sake of determinacy we present below only the simplest model example when the entire heat is removed through the shaft, and the bearing is heat-insulated.

The equation of heat conduction for a shaft in a system of coordinates rotating with the shaft (in which the shaft is motionless) is written in the form

$$\rho c \frac{\partial \theta}{\partial t} = \lambda \Delta \theta - \alpha \theta_m,\ \theta_m = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R \theta r dr d\psi,$$

where the Laplacian is calculated for the cylindrical coordinates r, ψ, where $d\psi = d\varphi - \Omega dt$, and the introduction of the term $\alpha\theta_m$ corresponds to one of the possible methods of semiempirical description of heat removal to external sinks.

On the shaft surface there occur pulsations of temperature and heat flux with frequency Ω_0. The depth of penetration of these pulsations into the shaft is, according to order of magnitude, equal to $(\lambda/\rho c\Omega_0)^{1/2}$, i.e., it is small when the angular frequency of the rotation is sufficiently large. It is physically clear that under real conditions the influence of such a "skin effect" on the deformation of the shaft is negligibly small. It is therefore natural that these pulsations are eliminated from the analysis and that time-averaging is carried out in the range from zero to $2\pi/\Omega_0$. Denoting by T the temperature θ thus averaged, we write the equation of heat conduction in the shaft

$$\rho c \frac{\partial T}{\partial t} = \lambda \Delta T - \alpha T_m, \ T_m = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R Trdrd\psi. \tag{20}$$

To formulate the boundary conditions, we have to examine the convective heat conduction in the layer of lubricant. Assuming for the sake of simplification that the Peclet number for this layer is small compared with unity (this complements in a natural manner the assumption that the Reynolds number for the layer of lubricant is small) and taking into account that the bearing is assumed to be heat-insulated, we have

$$\lambda' \frac{\partial^2 \theta'}{\partial n^2} + g = 0, \ \left. \frac{\partial \theta'}{\partial n} \right|_{n=h} = 0.$$

Integrating with respect to n in the interval (0, h), we find from this

$$\lambda' \left. \frac{\partial \theta'}{\partial n} \right|_{n=0} = \int_0^h g dn = G,$$

which determines the heat flux to the shaft surface. We obtain the corresponding boundary condition for Eq. (20) by averaging the last relation over time. Thus,

$$\lambda \left. \frac{\partial T}{\partial r} \right|_{r=R} = \frac{\Omega_0}{2\pi} \int_0^{2\pi/\Omega_0} G dt = \frac{1}{2\pi} \int_0^{2\pi} G d\varphi_0, \ \varphi_0 = \psi + \Omega_0 t, \tag{21}$$

where the argument φ of the integrand, with adequate choice of the beginning of the measurement of time, can be represented in the form

$$\varphi = \varphi_0 + \varepsilon z, \ z = \int_0^t \Omega_1 dt = C_z \cos(\Omega_0 t + \beta_z) \tag{22}$$

(this last relation follows directly from (19)).

We then calculate g and G taking into account the relations presented above; as before we retain only the principal terms concerning the ratio h/R and the terms not higher than of first order of powers of ε. Integrating

$$g = \frac{\mu}{2} \left(\frac{\partial u}{\partial n} \right)^2 = \frac{\mu}{2} \left[\frac{1}{2\mu R} \frac{\partial p}{\partial \varphi} (2n - h) - \frac{v_t}{h} \right]^2$$

with respect to dn in the interval (0, h), taking (4) and (8) into account, we obtain

$$G = \frac{\mu R^2}{h^3} \left\{ \frac{1}{2} [3(h - h_*)^2 + h^2] \Omega^2 + 6\varepsilon (h - h_*) \Omega \left(\frac{dx}{dt} \cos \varphi - \frac{dy}{dt} \sin \varphi \right) \right\}. \tag{23}$$

Without loss of accuracy we may write the value of h_*, determined in (10), in the form

$$h_* = h_*^* + \frac{2\varepsilon}{\Omega_0} \left(\frac{L_{c,3}}{L_c} \right)_0 \frac{dx}{dt}, \ h_*^* = \frac{L_2}{L_3} = \frac{2d(d^2 - e^2)}{2d^2 + e^2}, \tag{24}$$

and represent the value of G from (23) in one of the forms

$$G = G^* + \varepsilon \left(E_x \frac{dx}{dt} + E_y \frac{dy}{dt} \right) = G_0 + \varepsilon \left(D_x x + D_y y + D_z z + E_x \frac{dx}{dt} + E_y \frac{dy}{dt} + E_z \frac{dz}{dt} \right),$$
$$G^* = \frac{\mu R^2}{2h^3} [3(h - h_*^*)^2 + h^2] \Omega^2, \tag{25}$$

where h_*^* and G^* from (24) and (25) depend on e, Ω, and h with the same accuracy as the corresponding values of h_{*0} and G_0 in the unperturbed state of the radial bearing depend on e_0, Ω_0, and h_0 [6].

It is obvious that

$$G^*-G_0=\varepsilon\left[\left(\frac{\partial G^*}{\partial e}\right)_0 e_1+\left(\frac{\partial G^*}{\partial \Omega}\right)_0\Omega_1+\left(\frac{\partial G^*}{\partial h}\right)_0 h_1-e_0\left(\frac{\partial G^*}{\partial h}\right)_0(\sin\varphi_0)z\right]. \tag{26}$$

Taking (2), (3), and (22)-(26) into account, after some calculations we obtain for the coefficients of the representations in (25):

$$G_0=\frac{\mu R^2}{2h_0^3}\,[3(h_0-h_{*0})^2+h_0^2]\,\Omega_0^2,\quad D_x=-\left(\frac{\partial G^*}{\partial h}\right)_0\sin\varphi_0,$$

$$D_y=-\left(\frac{\partial G^*}{\partial h}\right)_0\cos\varphi_0-\left(\frac{\partial G^*}{\partial e}\right)_0,\quad D_z=-e_0\left(\frac{\partial G^*}{\partial h}\right)_0\sin\varphi_0,$$

$$E_x=\frac{6\mu R^2}{h_0^3}(h_0-h_{*0})\,\Omega_0\left[\cos\varphi_0-\left(\frac{L_{c,3}}{L_3}\right)_0\right],$$

$$E_y=-\frac{6\mu R^2}{h_0^3}(h_0-h_{*0})\,\Omega_0\sin\varphi_0,\quad E_z=\left(\frac{\partial G^*}{\partial \Omega}\right)_0,$$

$$h_{*0}=2d\,(d^2-e_0^2)\,(2d^2+e^2)^{-1}. \tag{27}$$

Retaining the previous designations, we must remember nevertheless that the coefficients in (27) are determined not only for the unperturbed state but that the argument φ, too, is replaced by φ_0, which corresponds to the replacement of φ by φ_0 in the representation of h_0 from (3). This, in particular, is connected with the appearance of the last term in the expansion (26).

Using the representations for x, y, and z introduced in (19) and (22), we can easily reduce G from (25) with the coefficients of (27) to a sum of terms containing as factors $\sin(\Omega_0 t+\beta)=\sin[\varphi_0-(\psi-\beta)]$ and $\cos(\Omega_0 t+\beta)=\cos[\varphi_0-(\psi-\beta)]$ with different β. Then, when we integrate the obtained expression in accordance with (21), we obtain as a result of simple transformations that

$$\lambda\left.\frac{\partial T}{\partial r}\right|_{r=R}=H+H_c\cos\psi+H_s\sin\psi, \tag{28}$$

where the coefficients H, H_c, and H_s are expressed through the integrals of the values of (27) with respect to $d\varphi_0$, and also through the amplitudes C and the phase shift angles β contained in (19) and (22), and they may be regarded as known; these coefficients are too cumbersome to be presented here.

The solution of the steady-state problem (21), (28) is trivial; we have

$$T=\frac{2H}{\alpha R}\left[1+\frac{\alpha}{4\lambda}\left(r^2-\frac{R^2}{2}\right)\right]+(H_c\cos\psi+H_s\sin\psi)\,\frac{r}{\lambda}, \tag{29}$$

from which it can be seen that with nonzero H_c, H_s the temperature distribution over the section of the shaft is, in fact, not axisymmetric, and thermal deformations therefore have to cause bending of the shaft, thus leading to the appearance of an effective centrifugal force. (In the unperturbed state the situation is completely different: the right-hand side of (28) in this case does not depend on ψ, and the temperature distribution is axisymmetric.)

The temperature field (29) relates to a simplier special situation. However, the general method of obtaining it remains the same in its basic fundamentals, even when the problem becomes more complicated in view of the actual features of heat removal in real systems.

To find a correlation between the distribution (29) and the amplitude A of the centrifugal force, we have to solve the problem of thermoelasticity, taking into account the existence of other rotating parts coupled with the shaft. It is clear that the result, which may be represented formally in the form

$$A = f(H, H_c, H_s), \tag{30}$$

yields a transcendental algebraic equation for A because the arguments of the function on the right-hand side of (30) are expressed through the amplitudes C of the representations (19) and (22), and the latter, in their turn, are proportional to A. The solution of this equation enables us to express the amplitude of the centrifugal force solely through the physical and the regime parameters of the system, and then, by using the methods suggested above, to find the characteristics of the oscillating motion of the shaft and the temperature distribution in it. The form of the function in (30) depends strongly on the special design features of the entire plant, and it has to be determined directly for a real plant.

In the analysis of flow in the layer of lubricant, in determining the dynamic response to the forces applied to the shaft, in calculating energy dissipation and finding the temperature field inside the shaft, we previously ignored completely the dependence of the viscosity of the liquid on the temperature. Under certain conditions this dependence may even lead to a qualitative change of the very nature of the flow, viz., to the appearance of a so-called "hydrodynamic thermal shock" [8, 9]. However, in most situations of practical importance the mentioned factor leads merely to some changes of the steady flow and the steady temperature field in the layer of lubricant which can be estimated with the aid of the standard method of the small parameter [5].

Thus the newly developed methods make it possible in principle to find a correlation between the oscillations of the shaft of a radial bearing and the perturbing force acting on it, between the perturbed flow in the layer of lubricant and these oscillations, and between the distribution of the shaft temperature and the characteristics of such a flow. Closing of the problem of unbalancing (which can be represented as the result of a peculiar thermohydrodynamic unstable system) is effected with the aid of the solution of the independent problem of thermoelasticity. Generalizing the results to real systems entails, after all, very cumbersome and laborious calculations but it does not encounter any difficulties of a fundamental nature.

NOTATION

A, amplitude of the centrifugal force; B, C, amplitudes of the variables x, y, z; c, specific heat; D, E, coefficients determined in (27); d, mean clearance; e, eccentricity; **F**, force acting on the shaft from the side of the layer of lubricant; G, g, functions introduced in (21); H, coefficients in (28); h, thickness of the layer of lubricant; h_*, value of (10); I, moment of inertia per unit length of the shaft; J, K, L, functions and constants determined in the text; M, external moment of forces; m, mass per unit length of the shaft; N, moment of forces acting on the shaft from the side of the layer of lubricant; ~~n–r–R~~; **P**, external force; p, pressure; Q, flow rate of the liquid; R, radius of the shaft; r, radial coordinate; T, averaged temperature; t, time; u, w, components of speed in the cylindrical system of coordinates; v_t, v_n, components of speed on the shaft surface; x, y, coordinates of displacement of the axis of the shaft; z, variable introduced in (22); α, heat-transfer coefficient; β, phase shift angles; ε, parameter of order; θ, θ', temperature in the shaft and in the layer of lubricant, respectively; λ, λ', thermal conductivities of the material of the shaft and of the liquid, respectively; μ, ν, dynamic and kinematic viscosity, respectively; σ, stress tensor; φ, φ_0, angular coordinates in the laboratory system; ψ, angular coordinate in the system conjugated with the shaft. Subscripts and superscripts: 0, values relating to the unperturbed state; 1, perturbations; *, functions whose dependence on the perturbed parameters has the same form as in the unperturbed state.

LITERATURE CITED

1. V. K. Zhitomirskii, Mechanical Oscillations and the Practice of Eliminating Them [in Russian], Moscow (1966).
2. F. M. Dimentberg, Flexural Vibrations of Rotating Shafts [in Russian], Moscow (1959).
3. M. E. Levit and V. P. Roizman, Vibration and Balancing of Aircraft Engine Rotors [in Russian], Moscow (1970).
4. A. S. Gol'din, M. I. L'vov, and A. V. Ur'ev, Energomashinostroenie, No. 5, 6-9 (1983).
5. M. A. Galakhov, P. B. Gusyatnikov, and A. P. Novikov, Mathematical Models of Contact Hydrodynamics [in Russian], Moscow (1985).

6. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Part 2, Moscow (1963).
7. É. L. Poznyak, Vestn. Mashinostr., No. 10, 29-34 (1965).
8. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, Dokl. Akad. Nauk SSSR, 163, No. 1, 133-136 (1965).
9. Yu. A. Buevich and M. I. Zaslavskii, Inzh.-Fiz. Zh., 42, No. 5, 813-819 (1982).

RADIANT HEAT TRANSFER IN A CLOSED SYSTEM OF SEMIOPAQUE BODIES SEPARATED BY AN EMITTING AND ABSORBING GAS MEDIUM

V. P. Gorshenin

UDC 536.3

A radiant heat-transfer problem is solved for a closed emitting system bounded by a nonisothermal semiopaque shell with the absorption and emission of a nonisothermal gas medium taken into account.

Analysis [1-4] shows that the solution of radiant heat-transfer problems at this time is performed for systems of bodies opaque to thermal radiation.

The extensive utilization of films and plastics in the construction of modern structures evokes the necessity to solve radiant heat-transfer problems for systems of semiopaque bodies relative to thermal radiation. Moreover, the space of these structures is filled with a nonisothermal medium emitting and absorbing thermal radiation since triatomic gases are usually contained therein. The cause of the nonisothermy of the gas space is the different temperature of its bounding surfaces.

We solve the problem formulated in general form first for boundary conditions of the first kind. As is known, in this case the temperatures of the body surfaces and the gas medium are given in this case. It is required to determine the resultant radiation Q_r of each of the elements of the emitting system.

Let us consider a nonisothermal semiopaque shell in which a nonisothermal medium is enclosed. In order to make an assumption about the diffuse nature of the emission, we divide the shell and the gas, respectively, into n isothermal surfaces and m = n + 1 isothermal spaces, as is shown in Fig. 1. The surfaces and the gas spaces are assumed gray.

Each of the n surfaces of the closed system has the temperature T_i and the following integral hemispherical radiation characteristics: ε_i, A_i, D_i. The temperature of the i-th space of the medium equals T_{gi} and its integral hemispherical radiation characteristics for the temperature T_{gj} and T_j have the respective values $\varepsilon_{gi,j}$, $a_{gi,j}$, $d_{gi,j}$ and $\varepsilon_{gi,j}$, $A_{gi,j}$, $D_{gi,j}$. Because there are no suspended particles in the gas medium we consider the energy scattering effect not to hold and, therefore, $d_{gi,j} = 1 - a_{gi,j}$; and $D_{gi,j} = 1 - A_{gi,j}$.

In connection with the fact that the surfaces and gas spaces are diffuse, we characterize the geometry of the body system by the mean angular coefficients φ_{j-i}, φ_{gj-i}, φ_{i-gj}, φ_{gi-gj}. The generalized angular coefficients are here determined by using the expression $\psi = D\varphi$, since the transmissivity D is taken out from under the integral.

According to Fig. 2, the resultant emission for each of the n semiopaque surfaces can be represented in the form

$$Q_{r,i} = Q_{ie,i} - Q_{ef,i} - Q_{t,i} = p_i Q_{ie,i} - Q_{ef,i}. \tag{1}$$

All-Union Central Scientific-Research and Design Institute "Gipronisel'prom", Orel. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 2, pp. 305-309, February, 1988. Original article submitted October 16, 1986.

0022-0841/88/5402-0223$12.50